EVEN GRACEFUL LABELING OF $C_m \cup P_n$ (even m)

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Abstract:
The graph $C_m \cup P_n$ is odd graceful if m is even (M.Ibrahim Moussa). proved that the graph $C_m \cup P_n$ is even graceful if m is odd with some certain conditions (T.Mahalakshmi Senthil Kumar, T.Abarna Parthiban and T.Vanadhi. In this paper we studied that the graph $C_m \cup P_n$ is even graceful if m is even with certain conditions.
Keywords: *Graceful labeling, graceful graph, even graceful labeling.*

1. Introduction
If the vertices are assigned values subject to certain conditions, it is known as graceful labeling. The study of graceful graph and graceful labeling methods were introduced by Rosa.

A function f is called graceful labeling of the graph G if f: V → {0, 1, 2, ..., 2q} is injection and the induced function f^* : E → {1, 2, ….. q} defined as $f^*(uv) = |f(u) – f(v)|$ is bijective. A graph which admits graceful labeling is called a graceful graph.

A graph G = (V (G), E (G)) is said to admit even graceful labeling if f: V(G) → {0, 1, 2, …..2q - 1} is injective and the induced function f^*: E(G) →{2, 4, 6, ……2q-2} defined as $f^*(uv) = |f(u) – f(v)|$ is bijective. A graph which admits even graceful labeling is called an even graceful graph.

2. Even graceful of $C_m \cup P_n$
Theorem 2.1: If k is an even positive integer and k > 2 and let m = k+2 , then the graph $C_m \cup P_n$ is even graceful for every n = m+2.

Proof: Let $V(C_m) = \{u_1, u_2, ….u_m\}$, $V(P_n) = \{v_1, v_2, ….v_n\}$ where $V(C_m)$ is the vertex set of the cycle C_m and $V(P_n)$ is the vertex set of the path P_n and let q = (n+m).For every u_i and v_i , the even graceful labeling functions $f(u_i)$ and $f(v_i)$ respectively are defined as follows:

Vertex labeling of C_m:

$f(u_1) = 0$; $f(u_2) = 2q-2$; $f(u_3) = 4$; $f(u_4) = 2q-4$; $f(u_5) = 4$; $f(u_6) = 2q-6$ ……etc., $f(u_i) = 2q – i$, when i is even, $f(u_i) = i – 1$ when i is odd.

Vertex labeling of P_n:

$$f(v_i) = \begin{cases} i, & i = 0, 2, 4, \dots \\ (q+2) - i, & i = 0, 2, 4, \dots \end{cases}$$

The first value of $f(v_i)$ denotes the labeling of the vertices which are lying below and the second value of $f(v_i)$ denotes the labeling of the vertices lying above of the path P_n.

Edge labeling of C_m:

$f^*(u_1u_2) = 2q-2$; $f^*(u_2u_3) = 2q-4$; $f^*(u_3u_4) = 2q-6$ ….. $f^*(u_mu_1) = 2q-m$

Edge labeling P_n:

$f^*(v_iv_{i+1}) = 2q-2m-p$ where p = 0, 2, 4,…

These conditions show that each component in the given graph has even graceful, the path is even graceful, the cycle with an even number of vertices is even graceful .We can prove that the vertex labels are distinct and all the edge labels are even numbers {2,4,6,…,2q-2}.

In this theorem, the conditions mentioned above for even values of k are used to prove distinct condition of edge labeling; otherwise it can't form graceful graph.

This theorem is illustrated by the following example.

Example 2.2:

When k = 4 then by definition m = 6 and n = 8

Let V (C_6) = { $u_1, u_2, u_3, u_4, u_5, u_6$}, $V(P_n) = V(P_{m+2}) = V(P_8) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ where $V(C_6)$ is the vertex set of the cycle C_6 and $V(P_8)$ is the vertex set of the path P_8 and q = n+m = 14. Number of vertices above the path is 4 and below the path is 4. The even graceful labeling functions $f(u_i)$ and $f(v_i)$ are given below.

The vertex labeling of cycle C_6 :
$f(u_1) = 0$; $f(u_2) = 26$; $f(u_3) = 2$; $f(u_4) = 24$; $f(u_5) = 4$; $f(u_6) = 22$
The vertex labeling of path P_8 :
$f(v_1) = 0$; $f(v_2) =16$; $f(v_3) = 2$; $f(v_4) = 14$; $f(v_5) =4$; $f(v_6) = 12$; $f(v_7) = 6$; $f(v_8) = 10$
The edge labeling function f^* is defined as follows:
$f^*(u_1u_2) = 26$; $f^*(u_2u_3) = 24$; $f^*(u_3u_4) = 22$; $f^*(u_4u_5) = 20$; $f^*(u_5u_6) = 18$; $f^*(u_6u_1) = 22$ and $f^*(v_1v_2) = 16$; $f^*(v_2v_3) = 14$; $f^*(v_3v_4) = 12$; $f^*(v_4v_5) = 10$; $f^*(v_5v_6) = 8$; $f^*(v_6v_7) = 6$; $f^*(v_7v_8) = 4$

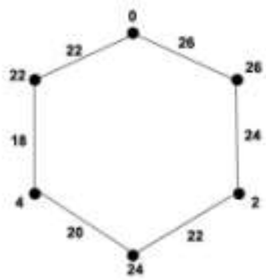
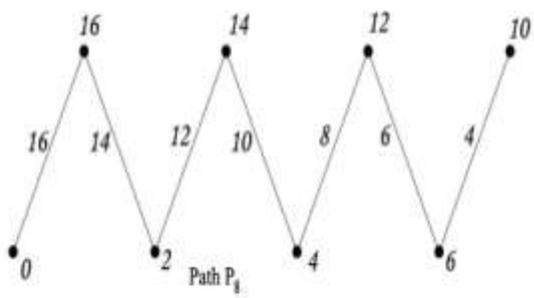


Fig : 1 Cycle C_6



Cleary, all the vertex labeling and edge labeling are even graceful and therefore the above graph is even graceful graph.

Example 2.3:

Consider the graph ($C_4 \cup P_6$) where m = 4 and q = 10 (because k = 2). Using theorem 2.1, the vertex labeling of the cycle is $f(u_1) = 0$; $f(u_2) = 18$; $f(u_3) = 2$; $f(u_4) =16$. The corresponding edge labeling is $f^*(u_1u_2) =18$; $f^*(u_2u_3) =16$; $f^*(u_3u_4) =14$; $f^*(u_4u_1)=16$

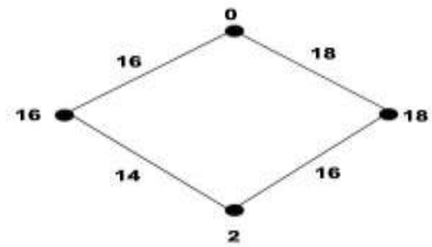
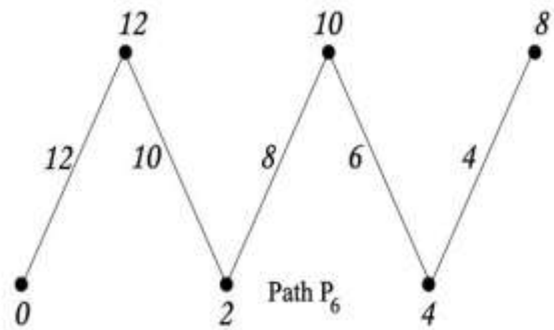


Fig : 2 Cycle C_4



clearly the above cycle is not graceful. Therefore the theorem is true for the positive even numbers of k only and k >2.

3. Conclusion:

In this paper, we explicitly proved a theorem for even graceful labeling of the graph ($C_m \cup P_n$) for m is even. We also found that this theorem is not true for m is odd.

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